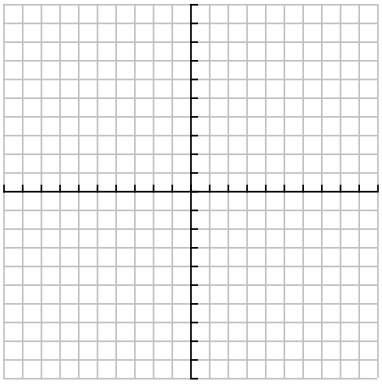
Parametric Equations

Sometimes, it is convenient to express both *x* and *y* as functions of a third variable, *t*. If $f(t)$ and $g(t)$ are both functions of *t*, where *t* is some interval of real numbers, then the equations $x = f(t)$ and $y = g(t)$ are called **parametric equations.** The variable t is called the **parameter.** If we think of *t* as time, then we know when each point of the graph is plotted.

Graphing Parametric Equations

1. Make a *t*, *x*, *y* table for the two equations.
2. Plot the ordered pairs of values of *x* and *y*.
3. Mark the **orientation** of the curve by using arrows to show the direction of the graph.

Example: Graph the parametric equations $x = t + 5$ and $y = 2t - 1$ for t in $[0,5]$.



Eliminating the Parameter

1. Set one equation equal to t.
2. Substitute that equation in for t in the other equation.
3. Sometimes it is more convenient to use a trigonometric identity to eliminate the parameter.

Examples: Eliminate the parameter and identify the graph of the parametric equation.

a) $x = 4t - 9,\ y = -t + 1,\ -\infty < t < \infty$

b) $x = 5\sin t,\ y = 5\cos t,\ -\infty < t < \infty$

Writing Parametric Equations for Line Segments

1. Write both parametric equations as linear functions: $x = m_1 t + b_1$, and $y = m_2 t + b_2$.
2. Substitute *x* and *t* values into the *x* equation to create a system of equations you can solve for m_1 and b_1.
3. Substitute *y* and *t* values into the *y* equation to create a system of equations you can solve for m_2 and b_2.

Examples:

Write parametric equations for the line segment starting at $(1,2)$ with $t = 0$ and ending at $(8,10)$ with $t = 1$.

Write parametric equations for the line segment starting at $(-2,4)$ with $t = 3$ and ending at $(5,-9)$ with $t = 7$.

Writing Parametric Equations for a Polar Equation

Use the equations $x = r\cos\theta$ and $y = r\sin\theta$. Replace *r* to obtain the parametric equations. When converting polar equations to parametric equations, θ acts as the parameter.

Example: Write parametric equations for the polar equation $r = 3\cos\theta$.